# Higgs mechanism and superconductivity in U(1) lattice gauge theory with dual gauge fields

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We introduce a U(1) lattice gauge theory with dual gauge fields and study its phase structure. This system is motivated by unconventional superconductors like extended $s$-wave and $d$-wave superconductors in the strongly-correlated electron systems. In this theory, the "Cooper-pair" field is put on links of a cubic lattice due to strong on-site repulsion between electrons in contrast to the ordinary $s$-wave Cooper-pair field on sites. This Cooper-pair field behaves as a gauge field dual to the electromagnetic U(1) gauge field. By Monte Carlo simulations we study this lattice gauge model and find a first-order phase transition from the normal state to the Higgs (superconducting) state. Each gauge field works as a Higgs field for the other gauge field. This mechanism requires no scalar fields in contrast to the ordinary Higgs mechanism.

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*Introduction.* – The Ginzburg-Landau (GL) theory has proved itself a powerful tool to describe the phase transitions of conventional $s$-wave superconductors. In field-theory terminology, the GL theory takes a form of Abelian Higgs model (AHM), and its phase structure has been studied by field-theoretical techniques and Monte Carlo (MC) simulations of lattice gauge theory. These studies are partly motivated by the work of Halperin, Lubensky, and Ma [1] which predicts a first-order phase transition. At present, it is established that the phase structure of three dimensional (3D) AHM on the lattice strongly depends on a parameter controlling fluctuations of the amplitudes $|\varphi(x)|$ of the Higgs (Cooper-pair) field [2]. At the London limit in which $|\varphi(x)|$ is fixed, there is only the confinement phase in the lattice model. As the fluctuations of $|\varphi(x)|$ are increased, a second-order phase transition to the Higgs phase appears, and for further fluctuations, the transition becomes of first-order.

Some strongly-correlated electron systems exhibit unconventional superconductivity (UCSC) [3] at low temperatures ($T$). The first $d$-wave superconductor $CeCu_2Si_2$ was discovered in 1979 [4]. In 1986, the cuprate high-$T_c$ superconductors were discovered [5], and later, it was found that they are $d$-wave superconductors. Thus, it is interesting to set up and study the GL theory of the UCSC. In the framework of weak-coupling theory, such studies have appeared [6]. However, the strong-coupling region remains to be studied [7]. In this Letter, we shall introduce a GL theory for the UCSC on a lattice, and study its phase structure by means of MC simulations. We shall see that the order parameter, a bilocal field, is regarded as a gauge field, and the knowledge and method of gauge theory are useful to study this GL lattice gauge theory. We find that this new type of gauge theory has a very interesting phase structure.

*Lattice gauge model for UCSC.* – Let us first consider a UCSC on a 3D spatial lattice. We put a "Cooper-pair field" $V_{xj}$ on the link $(x, x+j)$ of the lattice because of the strong on-site repulsion, where $x$ is the site index and $j(=1,2,3)$ is the direction index (it also denotes the unit vector of the $j$-th direction). $V_{xj}$ is related to electrons as

$$V_{xj} \propto \langle C^\dagger_{x\uparrow} C^\dagger_{x+j,\downarrow} - C^\dagger_{x\downarrow} C^\dagger_{x+j,\uparrow} \rangle, \tag{1}$$

where $C_{x\sigma}$ is the electron operator at $x$ with spin $\sigma = \uparrow, \downarrow$. In the rest of paper, we focus on the London limit of the $V_{xj}$ and put $|V_{xj}| = 1$, i.e., $V_{xj} = \exp(i\theta^v_{xj})$. At present it is believed that in the underdoped region of the high-$T_c$ superconductors the SC phase transition is a sort of the Bose-Einstein condensation of the Cooper-pair field $V_{xj}$, i.e., $V_{xj}$ has a finite amplitude and its phase fluctuation induces the SC phase transition.

There is another link field $U_{xj}$, a compact U(1) gauge field, describing the electromagnetic vector potential $\theta^u_{xj}$, $U_{xj} = e^{i\theta^u_{xj}}$. The original electron system is invariant under a local gauge transformation $C^\dagger_{x\sigma} \to e^{i\varphi_x} C^\dagger_{x\sigma}$. Under this transformation, $V_{xj}$ and $U_{xj}$ transform as

$$V_{xj} \to e^{i\varphi_{x+j}} V_{xj} e^{i\varphi_x}, \quad U_{xj} \to e^{i\varphi_{x+j}} U_{xj} e^{-i\varphi_x}. \tag{2}$$

By integrating over $C_{x\sigma}$ in path-integral method, one obtains the action $A_{\rm GL}$ of the GL theory, which is expressed in terms of two U(1) gauge fields $V_{xj}$ and $U_{xj}$. Because $A_{\rm GL}$ must respect the local gauge invariance under Eq.(2), it is straightforward to "derive" $A_{\rm GL}$ in the local expansion as

$$A_{\rm GL} = \frac{1}{2} \sum_{\rm pl} \Big[ c_u U^4 + c_v V^4 + c_m (UVUV + VUVU) + d_m (UUVV + 3 \text{ permutations}) \Big] + \text{c.c.}, \tag{3}$$

where $c_u$ etc. are effective parameters and some of them are increasing functions of $1/T$. Each term in $A_{\rm GL}$ is

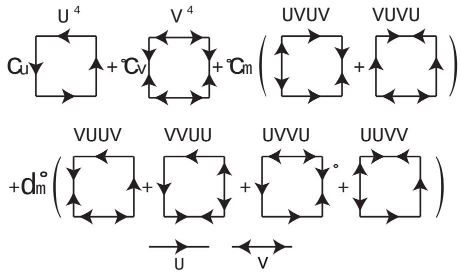


FIG. 1. Action $A_{\mathrm{GL}}$ of the GL theory (3).

depicted in Fig.1. For example, the $U^4$ term stands for the usual plaquette term $U^\dagger_{xi}U^\dagger_{x+i,j}U_{x+j,i}U_{xj}$ of lattice gauge theory. The partition function $Z$ is given by

$$Z = \int [dU][dV] \exp(A_{\mathrm{GL}}), \tag{4}$$

where $[dU] \equiv \prod_{xj} d\theta^u_{xj}/2\pi$, etc. We consider a 3D cubic lattice of the size $L^3$ with the periodic boundary condition. (The results given below are for $L = 16$ and 24.)

For vanishing $c_m$ and $d_m$, the two gauge fields $U_{xj}$ and $V_{xj}$ decouple with each other and the system reduces to two 3D pure U(1) gauge systems. As no phase transition takes place in the 3D U(1) pure gauge theory [8], the present system for $c_m = d_m = 0$ has only a single (confinement) phase. Then we assign various nonvanishing values to $c_m$ and/or $d_m$, and determine the phase structure in the $c_u - c_v$ plane for fixed $c_m$ and $d_m$. We note that the system has a symmetry $Z(c_u, c_v, c_m, d_m) = Z(c_v, c_u, c_m, d_m)$, corresponding to the interchange $U_{xj} \leftrightarrow V_{xj}$. Below we present the results for two typical cases: (i) $d_m = 0$ and $c_m > 0$ and (ii) $c_m = 0$ and $d_m < 0$.

*(i) Extended s-wave SC case ($d_m = 0$ and $c_m > 0$).* − To study the phase structure we measure the internal energy $E$ and the specific heat $C$ (fluctuation of $E$),

$$E \equiv -\langle A_{\mathrm{GL}}\rangle/L^3, \;\; C \equiv \langle (A_{\mathrm{GL}} - \langle A_{\mathrm{GL}}\rangle)^2\rangle/L^3. \tag{5}$$

We considered $d_m = 0, c_m = 0.2, 0.4, 0.6, 0.8, 1.0,$ and 1.2.

In Fig.2(a),(b) we show $E$ and $C$ for $d_m = 0, c_m = 0.6$ and $c_v/c_u = 0.1$. At $c_v \sim 1$, $E$ shows a hysteresis, which implies a first-order phase transition. The $c_m$ term works as a "Higgs coupling" of the "Higgs" field $V_{xj}(U_{xj})$ to the gauge field $U_{xj}(V_{xj})$ to stabilize their fluctuations and induce such a transition [9]. Thus the transition is expected from the confinement phase where $U_{xj}$ and $V_{xj}$ fluctuate violently to the Higgs (superconducting) phase where their fluctuations are small.

The data for $c_m \geq 0.6$ show signals of first-order phase transitions, while the data of $c_m \leq 0.4$ show no signals of phase transitions. In Fig.2(c) we show the phase diagram in the $c_u - c_v$ plane for $c_m = 0.6$. Similar phase diagram is obtained for $c_m = 0.8 \sim 1.2$.

To confirm the above interpretation of each phase, we measured instanton densities. We consider two kinds of

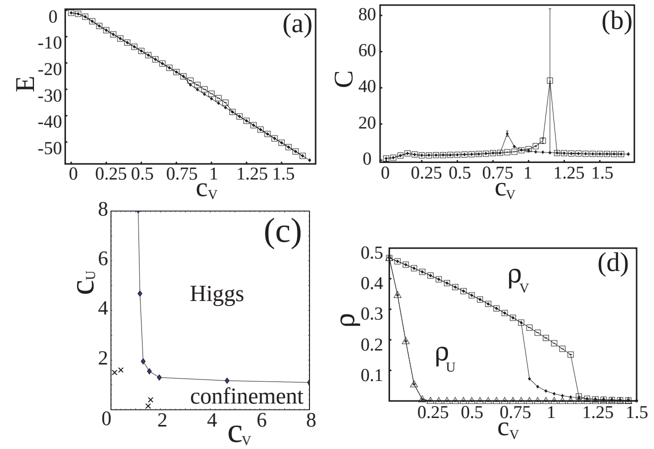


FIG. 2. Results for $d_m = 0, c_m = 0.6$. (a) Internal energy $E$ for $c_v/c_u = 0.1$ shows a hysteresis around $c_v \sim 1.0$, a signal of first-order transition. (b) Specific heat $C$ for $c_v/c_u = 0.1$ shows a small cusp at $c_v \sim 0.17$, which is interpreted as a crossover(see the text). (c) Phase structure determined by $E$ and $C$ for $L = 16$. The first-order transition line separates the confinement phase and the Higgs phase. The cross symbols denote crossover. (d) Instanton densities $\rho_U$ and $\rho_V$ for $c_v/c_u = 0.1$. $\rho_V$ exhibits a discontinuity at $c_v \sim 1.0$, while $\rho_U$ decreases rapidly at $c_v \sim 0.17$.

instantons, i.e., $U$-instantons and $V$-instantons, and denote their average densities per cube as $\rho_U$ and $\rho_V$, respectively. For the $U$-instantons we employ the definition given in Ref. [10], which measures magnetic fluxes emanating from each smallest cube. Similar gauge-invariant definition is possible for the $V$-instantons [11]. In Fig.2(d) we plot $\rho_U$ and $\rho_V$. $\rho_V$ shows a discontinuity at the first-order transition point $c_v \sim 1.0$ just like $E$, and decreases very rapidly as $c_v$ increases. On the other hand, $\rho_U$ decreases very rapidly and almost vanishes for $c_v > 0.2$. This result indicates that the small cusp in $C$ of Fig.2(b) at $c_v \sim 0.17$ reflects a crossover from the dilute to dense instanton "phases" of the gauge field $U_{xj}$ [8,10,12]. Then we conclude that the system changes from the confinement phase to the Higgs phase as $c_v$ ($c_u$) increases.

In order to support the above conclusions, we also calculated expectation values of the $U$ and $V$-Wilson loops,

$$W_U(\Gamma) = \langle \prod_\Gamma U \rangle, \;\; W_V(\Gamma) = \langle \prod_\Gamma V \rangle, \tag{6}$$

where $\Gamma$ is a closed loop on the lattice, and the products of $U_{xj}$ and $V_{xj}$ in Eq.(6) are formed to be gauge-invariant. From the above results of instanton densities, we expect that $W_{U(V)}(\Gamma)$ obey the are law in the instanton-plasma phase for small $c_{u(v)}$ and the perimeter law in the instanton-dipole phase for large $c_{u(v)}$. In calculating $W_{U(V)}(\Gamma)$, we consider various shapes of $\Gamma$. For example, we take $\Gamma$'s having a fixed area and various perimeters, and vice versa. In Fig.3 we show the results for $c_m = 0.6$. In the case $c_u = c_v = 1.4$, $W_U(\Gamma)$ fits the area law, while the case $c_u = c_v = 2.0$ fits the perimeter

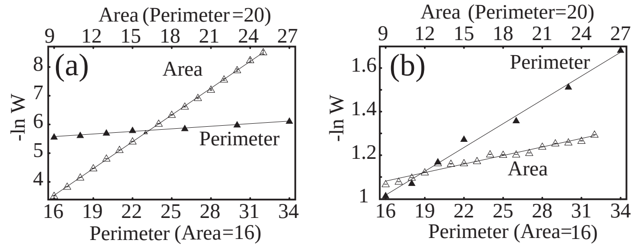


FIG. 3. Wilson loops for $d_m = 0, c_m = 0.6$ with $c_u = c_v$, which means $W_U(\Gamma) = W_V(\Gamma)$. (a) The case $c_u = c_v = 1.4$ fits the area law. (b) The case $c_u = c_v = 2.0$ fits the perimeter law rather than the area law. Fig.2(c) indicates that the transition between the two laws occurs at $c_u = c_v \sim 1.6$.

law. These results support the previous conclusion obtained from instanton densities.

Because all the coefficients in $A_{\rm GL}$ are positive in the present case, we expect that the observed SC state is an *extended* $s$-wave superconductor. We calculated $\langle UUVV\rangle$ in order to verify this expectation for $c_m \geq 0.6$ and $d_m = 0$ cases and found that $\langle UUVV\rangle$ has vanishing value in the normal state whereas it has a finite value in the Higgs phase showing hysteresis loop. Furthermore its value takes a negative as well as positive value depending on samples. Turning on a small but finite positive $d_m$ term, $\langle UUVV\rangle$ becomes positive. This result indicates that positive $d_m$ term is necessary to produce a genuine *extended* $s$-wave superconductor.

*(ii) d-wave SC case ($c_m = 0$ and $d_m < 0$).* — We expect and verified that the $d_m$-term with $d_m < 0$ enhances $d$-wave condensation of $V_{xj}$; $\Delta_{ij} \equiv \langle U_{xi}U_{x+i,j}V^\dagger_{x+j,i}V_{xj}\rangle < 0$. For large $c_u$, the fluctuations of $U_{xj}$ are suppressed as $U_{xj} \sim 1$ [up to Eq.(2)], hence the $d$-wave configuration $\langle V^\dagger_{xi}V_{x+i,j}\rangle < 0$ ($i \neq j$) is preferred, although there are no configurations with all negative $\Delta_{ij}$. We considered the cases $d_m = -0.4, -0.6, -0.8, -1.0$, and measured $E$, $C$, $\rho_U$, and $\rho_V$ as before. No signals of phase transitions are found for $d_m = -0.4$ and $-0.6$, whereas signals of phase transitions to the $d$-wave superconducting phase are obtained for $d_m < -0.6$.

In Fig.4(a),(b) we present $E$ and $\rho$ for $c_u/c_v = 1.0$ and $d_m = -0.8$. $E$ and $\rho$ show three first-order phase transitions along $c_v = c_u$. In Fig.4(c) we present the phase structure. There are four phases (I)-(IV). The phase (I) is the confinement phase. (II) is the "staggered state" which is generated by the frustration of strong negative $d$-term. It breaks the translational symmetry by the unit lattice spacing as supported by the Wilson loop of Fig.4(d) which has two branches, one for even areas and one for odd areas. (III) is the disorder state connecting (II) and (IV). (IV) is the Higgs phase corresponding to $d$-wave superconductor. These interpretations are consistent with the behavior of $\rho$ in Fig.4(b). The measurement of $\Delta_{\mu\nu}$ shows that the cubic symmetry $\Delta_{23} = \Delta_{12} = \Delta_{13}$ is maintained in (IV), whereas it is reduced to the square

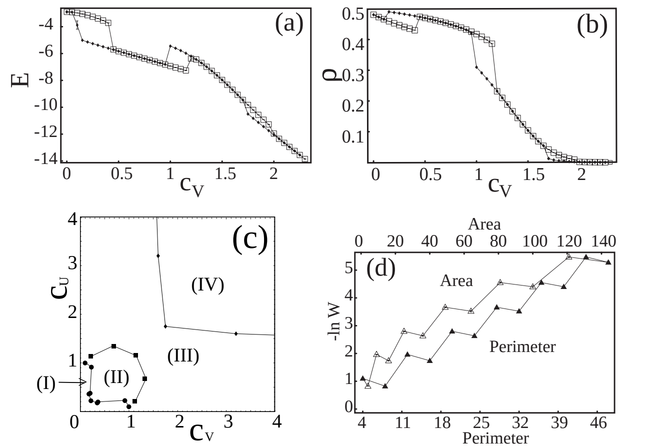


FIG. 4. Results for $c_m = 0, d_m = -0.8$. (a) $E$ and (b) $\rho_U = \rho_V$ for $c_u/c_v = 1.0$. They show three hysteresis loops. (c) Phase structure in the $c_u - c_v$ plane. (d) Wilson loop of $M \times M$ square for $c_u = c_v = 0.9$ shows two smooth branches for even $M$ and odd $M$, i.e., staggered perimeter law.

symmetry, e.g., $\Delta_{23} < \Delta_{12} = \Delta_{13}$ in (II).

We also studied an anisotropic system in which the intraplane $d_m$ term is larger than the interplane $d_m$ term because many of the real materials have a layered structure. In Fig.5 we show $E$ and the instanton density for $c_u/c_v = 1.0$, $c_m = 0$, intraplane $d_m = -1.0$ and interplane $d_m = -0.8$. There is a first-order phase transition near $c_u = c_v = 1.7$, whereas the other transitions existing in the isotropic case in Fig.4 disappeared. Thus the phase (I) and (II) disappear whereas the two phases (III) and (IV) survive. This phase transition in the anisotropic case should correspond to the SC transition observed in the heavy-fermion materials, etc.

*Quantum phase transition.* — We have considered the gauge model $A_{\rm GL}$ (3) defined on the cubic lattice and studied its phase structure. The phase transitions which we found in the previous discussion correspond to *thermal phase transitions*, i.e., the coefficients in $A_{\rm GL}$ are increasing functions of $1/T$ and the SC phase appears as $T$ is lowered. Recently, in the studies of the strongly-correlated electron systems like the high-$T_c$ cuprates and the heavy-fermion materials, significance of *quantum*

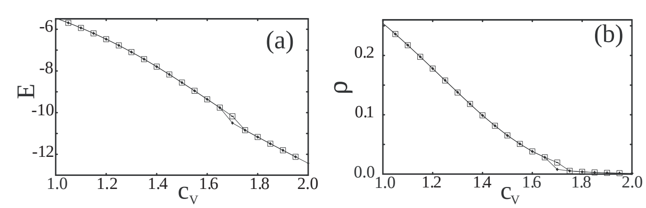


FIG. 5. (a)Internal energy for an anisotropic system. Parameters are $c_u/c_v = 1$, $c_m = 0$, intraplane $d_m = -1.0$ and interplane $d_m = -0.8$. There is a first-order phase transition near $c_u = c_v = 1.7$. (b)Instanton density also shows hysteresis loop near $c_u = c_v = 1.7$.

*phase transition* (QPT) has been recognized [13]. In particular, the QPT in ordinary $s$-wave charged SC was studied by using a XY model coupled with a U(1) gauge field [14] and very recently this model was applied for the QPT in the high-$T_c$ cuprates in order to explain the anormalous behavior of the superfluid density near the quantum critical point (QCP) at $T = 0$ [15].

Quantum theory of the present GL theory can be constructed straightforwardly. In the continuum imaginary-time formalism, $U_{xj}$ and $V_{xj}$ depend on the imaginary time $\tau$. Then we discretize the imaginary-time axis and define the quantum GL theory on the 4D hypercubic lattice. The action $A^{\rm q}_{\rm GL}$ of the quantum system has a similar form of $A_{\rm GL}$ in Fig.1, but it is defined on the 4D lattice. It involves the time component $U_{x\tau}$ but does *not* contain the terms including $V_{x\tau}$ as the Cooper-pair field lives on links of the *spatial* lattice.

We also studied this quantum system $A^{\rm q}_{\rm GL}$ by the MC simulations. In the practical experiments, external conditions and properties of samples like external pressure, doping parameter, etc., change the effective parameters contained in $A^{\rm q}_{\rm GL}$. Here we consider typical two cases in which the coefficients in $A^{\rm q}_{\rm GL}$ are scaled as $(c_u, c_v, c_m, d_m) = g(1, 1, 1, 0)$ and $g(1, 1, 1, -1)$ where $g$ is a positive parameter. In Fig.6, we show $E$ and the instanton density for the former case. The result shows a first-order phase transition from the normal (or insulating) phase to the SC phase. We found that the other case also has a similar phase structure. This result should be compared with the observed SC phase transition at very low $T$'s in various heavy-fermion materials, etc.

*Conclusion.* – In this paper, we considered a GL theory for UCSC by introducing a "Cooper-pair" field on spatial links. We showed that the GL theory can be regarded a new type of lattice gauge model which contains two kinds of gauge fields. By means of MC simulations, we clarified its phase structure. For the case of $c_m, d_m > 0$, there are two phases for sufficiently large $c_m$, which correspond to the normal and SC (Higgs) phases. They are separated by a first-order phase transition line. On the other hand, for the case of $d_m \leq -0.8$ there are four phases in the isotropic case. Only the two of them survive in the anisotropic case which corresponds to the layered structure. The observed phase transitions should be compared with the experiments of the high-$T_c$ cuprates and heavy-fermion materials.

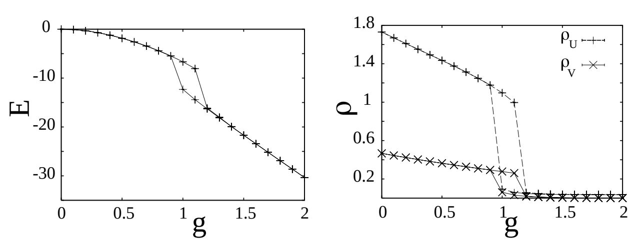


FIG. 6. $E$ and instanton density of the quantum system $A^{\rm q}_{\rm GL}$ with $(c_u, c_v, c_m, d_m) = g(1, 1, 1, 0)$. They exhibit first-order phase transition.


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[7] Recently, M.N. Chernodub, E.-M. Ilgenfritz, and A. Schiller, Phy. Rev. **B73**, 100506(2006), studied a related gauge theory, where a doubly-charged scalar field on sites is introduced instead of the $V$-field in the present paper.

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[11] To define the $V$-instantons we consider the replacement $\varphi_x \to -\varphi_x$ for the odd sites in Eq.(2). Then $V_{x\mu}$ transforms just like $U^\dagger_{x\mu}$ before the replacement, so it is straightforward to define the $V$-instantons.

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